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Günter W. Beck and Volker Wieland
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Prof. Volker Wieland, Ph.D.
Money in Monetary Policy Design under Uncertainty: A Formal Characterization of ECB-Style Cross-Checking*

Günter W. Beck¹ and Volker Wieland²

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Abstract:
The European Central Bank has assigned a special role to money in its two pillar strategy and has received much criticism for this decision. The case against including money in the central bank’s interest rate rule is based on a standard model of the monetary transmission process that underlies many contributions to research on monetary policy in the last two decades. In this paper, we develop a justification for including money in the interest rate rule by allowing for imperfect knowledge regarding unobservables such as potential output and equilibrium interest rates. We formulate a novel characterization of ECB-style monetary cross-checking and show that it can generate substantial stabilization benefits in the event of persistent policy misperceptions regarding potential output.

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¹ Goethe University Frankfurt and CFS; E-mail: gbeck@wiwi.uni-frankfurt.de

² Goethe University Frankfurt, CEPR and CFS
Corresponding author: Goethe University Frankfurt, Mertonstrasse 17, D-60325 Frankfurt. Tel.: ++49-69-79825288. E-mail: wieland@wiwi.uni-frankfurt.de
1 Introduction

Contrary to the monetary policy strategies of the U.S. Federal Reserve and many inflation-targeting central banks, which assign no special role to monetary aggregates, the European Central Bank has maintained a separate and important role for money in its two pillar strategy. The ECB distinguishes an ‘economic’ and a ‘monetary’ pillar:1

- **Economic analysis** assesses the short to medium-term determinants of price developments. The focus is on real activity and financial conditions in the economy. The economic analysis takes account of the fact that price developments over those horizons are influenced largely by the interplay of supply and demand in the goods, services and factor markets.

- **Monetary analysis** focuses on a longer-term horizon than the economic analysis. It exploits the long-run link between money and prices. The monetary analysis mainly serves as a means of cross-checking, from a medium to long-term perspective, the short to medium-term indications for monetary policy coming from the economic analysis.”

In terms of economic theory, the long-run link noted by the ECB is related to the equation of exchange, that is, the definition of the velocity of money. Rewritten in growth terms it relates money growth, inflation and output growth to the change in velocity. In the long-run, once output growth and the change in velocity have settled down to trend, the equation of exchange implies a proportional relationship between money growth and inflation. In terms of empirics, this relationship has manifested itself most clearly in periods of very high inflation. Recent empirical assessments, however, have re-emphasized its validity in periods of moderate to low inflation in leading industrial economies.2

On this basis, Gerlach (2003, 2004) has proposed to augment the standard Phillips curve, which accounts for shorter-term inflation dynamics, resource utilization gaps and inflationary shocks, with a measure of long-run or low-frequency money growth. Such an augmented Phillips curve unifies the two pillars of the ECB in a single assessment of inflationary risks, and—if treated as a structural relationship—provides a rationale for including filtered money growth in the central bank’s optimal interest rate rule. The ECB’s description of its strategy, however, does not rely on a direct effect of money on inflation in the Phillips curve. Rather, it focuses on the long-run link and its usefulness for identifying medium- to long-term inflationary risks. Thus, we aim to develop an alternative rationale.

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for including money in the policy rule that stays as close as possible to the ECB’s stated reasons.3

We formally characterize ECB-style cross-checking using a policy rule with two components. The first component aims to control inflationary risks based on a standard Phillips curve and aggregate demand relationship. Essentially, it is the optimal interest rate rule of an inflation-targeting central bank. If implemented successfully this rule should ensure that inflation averages around the central bank’s inflation target. Its weakness is that it relies on knowledge of unobservables such as the equilibrium real interest rate and potential output that may be subject to large and persistent policy misperceptions.

The second component captures the idea of cross-checking using the long-run relationship between money and inflation. We assume that the central bank checks regularly whether a filtered money growth series adjusted for output and velocity trends averages around the inflation target. If the central bank obtains successive signals of a sustained deviation of inflation from target it adjusts interest rates accordingly.

Our simulations indicate that persistent policy misperceptions regarding potential output induce a policy bias that translates into persistent deviations of inflation and money growth from target. In this case, our “two-pillar” policy rule may effectively overturn the policy bias. Cross-checking relies on filtered series of actual money and output growth without requiring estimates of potential output. Indirectly, however, it helps the central bank to learn the proper level of interest rates.

2 Money growth and inflation in the long run

The equation of exchange defines velocity, \( v_t = -m_t + p_t + y_t \), where \((m, y, p)\) denote the logarithms of money, output and the aggregate price level. Taking first differences we approximate the equation in growth terms:

\[
\Delta v_t = -\Delta m_t + \Delta p_t + \Delta y_t.
\]

\(\Delta\) is the first-difference operator. In the long-run, output growth and the change in velocity will settle down to trend and reveal a proportional relationship between money growth and inflation. In the short-run, however, fluctuations in velocity and output growth are likely to obscure this relationship. The behavior of velocity may be characterized as a function of the nominal interest rate \(i\), real output and

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3See, e.g., Issing (2005) for a detailed discussion of the function that the monetary pillar plays for the conduct of the ECB’s monetary policy.
money demand shocks $\varepsilon_{md}$ using a standard money demand equation:

$$m_t - p_t = \gamma_y y_t - \gamma_i i_t + \varepsilon_{md}.$$  \hfill (2)

Here, $\gamma_y$ denotes the income elasticity and $\gamma_i$ the semi-interest rate elasticity of money demand. Money demand shocks are assumed to be normally distributed with mean zero and variance $\sigma^2_{md}$. Taking first differences, re-arranging (2) and combining with (1) we obtain:

$$\Delta v = (1 - \gamma_y) \Delta y + \gamma_i \Delta i + \Delta \varepsilon_{md}.$$  \hfill (3)

Long-run equilibrium values (superscript *) can then be determined as follows. In the long-run, money demand shocks would average to zero, and the nominal interest rate would settle down to its steady state level. Thus, the long-run trend in velocity corresponds to $\Delta v^* = (1 - \gamma_y) \Delta y^*$, and long-run inflation is proportional to long-run money growth adjusted for output and velocity trends:

$$\Delta p^*_t = \Delta m^*_t - \gamma_y \Delta y^*_t.$$  \hfill (4)

Recent studies obtained empirical support for this long-run relationship using various filters or frequency-specific estimation. And more interestingly, they have found money growth to lead inflation at this frequency. To give an example, Gerlach (2004) uses the following filter

$$\mu_f^t = \mu_f^t - 1 + \lambda (\mu_t - \mu_f^t - 1),$$  \hfill (5)

to approximate long-run values of inflation and money growth. In his work, $\mu_t$ may alternatively stand for money growth $\Delta m_t$ or money growth adjusted for output growth. In our paper we will follow equation (4) and adjust money growth using the estimate of the income-elasticity of money demand, i.e.

$$\mu_f^t = \Delta m_f^t - \gamma_y \Delta y_f^t.$$  \hfill (6)

### 3 Monetary policy design without money

Most research on monetary policy rules in the last two decades has focused on models, in which the monetary transmission mechanism works as follows: the nominal interest rate affects the real interest rate due to price rigidity, the real rate influences the output gap via aggregate demand and the output gap impacts on

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A trend in velocity may not only arise from potential output growth $\Delta y^*$ with an income elasticity $\gamma_y$ different from unity, but also from other sources such as financial innovations (see Orphanides and Porter (2001) and Masuch, Pill and Willeke (2001)).
inflation via a standard Phillips curve. Thus, monetary aggregates play no direct role in the transmission of policy from nominal interest rates to inflation. Money supply instead is determined recursively from a money demand equation.

To illustrate this basic point we use a simple New-Keynesian style model with backward-looking expectations in the spirit of Svensson (1997) and Orphanides and Wieland (2000). The model consists of a Phillips curve and an aggregate demand equation:

\[ \pi_t = \pi_{t+1}^e + \alpha_y (y_t - y_t^e) + \epsilon_{\pi_t}, \]  
\[ y_t - y_t^e = (y_{t+1}^e - y_{t+1}^{e,\pi}) - \beta_r (i_t - \pi_{t+1}^e - r_t^e) + \epsilon_{\pi_t}, \]  

where \( \pi_t = \Delta p_t \) denotes inflation, \((\epsilon_{\pi_t}, \epsilon_{\pi_t})\) stand for zero-mean cost-push and demand shocks respectively with variances \((\sigma_{\pi_t}^2, \sigma_{\pi_t}^2)\), \(r^e\) denotes the long-run equilibrium interest rate and the superscript \(e\) refers to market expectations, which we assume to be backward-looking.

An inflation-targeting central bank would set the nominal interest rate \(i_t\) in order to minimize expected discounted inflation deviations from target:

\[ \min_{i_t, i_{t+1}, \ldots} E_t \left\{ \sum_{s=t}^{\infty} \delta^{s-t} (\pi_s - \pi^*)^2 \right\}, \]  

where \(\pi^*\) denotes the central bank’s inflation target and \(\delta\) its discount factor. Consequently, optimal monetary policy corresponds to a Taylor-style interest rate rule, which responds to lagged inflation and output gaps but not to money growth:

\[ i_{t+1}^{opt} = r^e_t + \pi_{t-1} + \frac{1}{\alpha_y \beta_r} (\pi_{t-1} - \pi^*) + \frac{1}{\beta_r} (y_{t-1} - y_{t-1}^e). \]  

The superscript ‘opt’ refers to ‘optimal’.

To be clear, the central bank achieves the desired interest rate setting by conducting open-market operations that influence the money supply. Thus, the money supply is determined according to the money demand equation (2) consistently with the desired policy rate, current output and the price level. However, money does not appear as a variable in the central bank’s optimal interest rate rule.

Inspired by the evidence for the long-run relationship between money and inflation, Gerlach (2003, 2004) proposed to include a filtered measure of money growth or adjusted money growth in the estimation of the short-run Phillips curve. If a central bank were to consider this empirical two-pillar Phillips curve as a structural relationship it would conclude that a measure of filtered money growth
should enter in its interest rate rule.\footnote{We develop this argument further in the working paper version (Beck and Wieland (2006)).} Such a rule could be viewed as an interpretation of the ECB’s two-pillar strategy. However, the ECB’s description of its strategy does not rely on a direct effect of money on inflation in the Phillips curve. Rather, it uses the monetary pillar to accumulate evidence signalling trend changes in inflation.

### 4 ECB-style cross-checking and policy design

We develop a characterization of ECB-style cross-checking that stays as close as possible to the ECB’s own description. Our proposed interest rate rule has two components:

\[
i_{t}^{CC} = i_{t}^{EA} + i_{t}^{MA} \tag{11}
\]

Here the superscript $CC$ refers to cross-checking, $EA$ to the interest rate setting implied by the ECB’s ‘economic analysis’ and $MA$ to an additive adjustment in interest rate setting that arises from the ECB’s ‘monetary analysis’. We set the first component equal to the optimal interest rate rule in the baseline model:

\[
i_{t}^{EA} = i_{t}^{opt} \tag{12}
\]

This interest rate setting should ensure that inflationary risks based on a standard Phillips curve are controlled perfectly and inflation fluctuates randomly around the mean, $\pi^\ast$. However, this component relies on knowledge of unobservables such as the equilibrium real interest rate, $r^\ast$, or potential output, $y^\ast$, that may be subject to large and persistent policy misperceptions.

The second component, $i_{t}^{MA}$, is novel and captures the idea of cross-checking using the long-run relationship between money and inflation. This component is additive and persistent, because it is intended to offset persistent policy biases due to imperfect information. We assume that the central bank regularly tests whether filtered and adjusted money growth, $\mu_f$, still averages around the inflation target. Thus, the central bank computes the normally-distributed test statistic,

\[
\kappa = \frac{\mu_{t-1} - \pi^\ast}{\sigma_{\mu_f}}, \tag{13}
\]

and checks whether $\kappa$ deviates from a critical value $\kappa^{crit}$. $\sigma_{\mu_f}$ denotes the standard deviation when $i_{t}^{EA} = i_{t}^{opt}$ is implemented with correct values of potential output and the mean of $\mu_f$ corresponds to $\pi^\ast$. If the central bank obtains successive signals of a sustained deviation from target, i.e. $(\kappa > \kappa^{crit}$ for $N$ periods) or $(\kappa < \kappa^{crit}$ for $N$ periods), it can make the necessary adjustments.
−κ^crit \text{ for } N \text{ periods}), it responds by adjusting interest rates accordingly. \footnote{The response coefficient on inflation deviations from target is the same as in } i_{t}^{opt}, namely \frac{1}{\alpha_{y} \beta_{r}}.\footnote{As long as } i_{t}^{MA} = i_{t}^{opt} \text{ is implemented with full knowledge of potential output, } y_{t}^{*} \text{ and the real equilibrium rate, } r^{e}, cross-checking with regard to } i_{t}^{MA} \text{ will almost never lead to an adjustment in interest rates. With imperfect knowledge, however, cross-checking may occasionally have very important effects on policy.}\footnote{The two parameters of } i_{t}^{MA}, \kappa^{crit} \text{ and } N \text{ play different roles. } \kappa^{crit} \text{ reflects the probability that an observed deviation of } \mu^{t} \text{ from } \pi^{*} \text{ is purely accidental (for example a 5\% or 1\% significance level). } N \text{ defines the number of successive deviations in excess of this critical value. Thus, the greater } N \text{ the longer the central bank waits to accumulate evidence of a sustained policy bias. For example, if } \kappa^{crit} \text{ is set to the 1\% critical value for the normal distribution (2.575) and the critical number of periods of sustained deviations } N \text{ is set to 4, the probability of such an event in the absence of policy misperceptions would be less than } 10^{-8}.\footnote{For example, Orphanides et al. (2000) estimate a process of misperceptions with a near unit root (0.96) and standard deviation of 3.77\%.}

\begin{align*}
i_{t}^{MA} = \begin{cases} i_{t-1}^{MA} + \left( \frac{1}{\alpha_{y} \beta_{r}} \right) (\mu_{t-1}^{f} - \pi^{*}) & \text{if } \kappa > \kappa^{crit} \text{ or } \kappa < -\kappa^{crit} \text{ for } N \text{ periods} \\
i_{t-1}^{MA} + 0 & \text{else} \end{cases} \tag{14}\end{align*}

As long as } i_{t}^{MA} = i_{t}^{opt} \text{ will almost never lead to an adjustment in interest rates.}

\section{5 Cross-checking and policy misperceptions}

Recent research exploiting data on historical revisions to real-time estimates of the output gap has identified very persistent policy misperceptions. \footnote{The response coefficient on inflation deviations from target is the same as in } i_{t}^{opt}, namely \frac{1}{\alpha_{y} \beta_{r}}. The persistence of measurement errors arises primarily from biased estimates of unobservable potential output. Thus, if a central bank relies on potential output measures in policy design, its policy stance may be biased for a sustained period of time. To illustrate this effect we define the policymaker’s estimate of potential output, \hat{y}_{t}^{*} = y_{t}^{*} + \text{bias}_{t}, as the sum of true potential output and a measure of the misperception denoted by \text{bias}_{t}, and include it in the baseline rule:

\begin{align*}
i_{t}^{opt} = r_{t}^{opt} = r_{t}^{*} + \pi_{t-1} - \pi^{*} + \frac{1}{\alpha_{y} \beta_{r}} (\pi_{t-1} - \pi^{*}) + \frac{1}{\beta_{r}} (y_{t-1} - y_{t-1}^{*} - \text{bias}_{t-1}). \tag{15}\end{align*}

The resulting bias in interest rate policy will induce a persistent deviation of inflation from target. For example, if the central bank’s estimate of potential output were to remain permanently 1\% above its true level (i.e. \text{bias}_{t} = 1\% \text{ for all } t), average inflation would increase by \left( \alpha_{y} \beta_{r} \right) \left( \beta_{r} \right)^{-1} \text{ percentage points.}

To illustrate this point further we calibrate the model defined by equations (7), (8) and (2) (see Beck and Wieland (2006)) and simulate the interest rate rule
(15) with the following sequence of misperceptions:

\[
\begin{align*}
\text{for } t &= (1,10) \quad \text{bias}(t) = 0 \\
\text{for } t &= (11,12,13,14) \quad \text{bias}(t) = (1,2,3,4) \\
\text{for } t &= (15,100) \quad \text{bias}(t) = 4 \\
\text{for } t &= (101,102,103) \quad \text{bias}(t) = (3,2,1) \\
\text{for } t &= (104,200) \quad \text{bias}(t) = 1
\end{align*}
\]

The central bank’s initial estimate of potential output is assumed to coincide with the true value. In periods 11 to 14 the central bank begins to overestimate potential output leading to a bias of 4% from period 14 onwards. From period 100 onwards the central bank’s overestimate of potential output declines to 1%.

Figure 1: Misperceptions, Money-Inflation Link and Cross-Checking

The top row of four panels in Figure 1 presents a simulation of the consequences of these misperceptions given a single draw of normally-distributed cost-push, demand and money demand shocks. The resulting, persistent increase in inflation cannot immediately be seen from current realizations but is eventually revealed by the filtered measures of inflation, \(\pi^f\), and money growth, \(\Delta m_f\). It amounts to 2% because the calibrated value of \(\alpha_y\) is 0.5.

This simulation illustrates the long-run relationship between money growth and inflation, and emphasizes the weakness of the policy rule, \(i_t^{EA} = \bar{r}^{opt}\), in the event of persistent misses on potential output. A similar effect would arise from incorrect estimates of the equilibrium real interest rate \(r^*\).
Of course, one may argue that the central bank will learn from its mistakes. Thus, we proceed to show that cross-checking as defined by the rule in equation (14) provides an effective avenue for learning and correcting the central bank’s policy bias. We repeat the preceding simulation using the cross-checking rule, $i_t^{CC}$ defined by (14) which includes an additive and persistent adjustment in the event of sustained deviations of filtered (adjusted) money growth from target. The outcome is reported in the second row of panels in Figure 1. We have dropped the panel with actual money growth, $\Delta m$, and have instead included a panel reporting the bias in the central bank’s estimate of potential output, $bias_t$, and the adjustment in interest rates due to cross-checking. This adjustment corresponds to $i_t^{MA}$ as defined in equations (14) and (13). The cross-checking rule responds to the increase in filtered money growth $\mu_{t-1}^f$ fairly quickly after the policy bias has arisen. The interest rate adjustment of $(\frac{1}{\alpha_y})(\mu_{t-1}^f - \pi^*)$ almost perfectly offsets the policy bias arising from potential output, $(\frac{1}{\beta_r})(bias_{t-1})$. Once the misperception of potential output declines after period 100, cross-checking soon leads to another adjustment of interest rates.\footnote{To assess the sensitivity of our findings we performed several Monte Carlos exercises. We drew 1000 series of shocks of length 200 from a normal distribution and considered the performance of our cross-checking rule under alternative parameter settings for $\lambda$ and $\kappa^{crit}$. We found that, on average, cross-checking leads to the appropriate interest rate adjustments offsetting the policy bias due to output gap misperceptions. The results are reported in Beck and Wieland (2006).}

6 Outlook

We have presented the first formal characterization of ECB-style cross-checking. Under the unrealistic assumption that the true values of potential output are known to the central bank our specification of cross-checking would never come into play. However, with imperfect knowledge there is a possibility of policy misperceptions. These misperceptions may generate sustained deviations of inflation from target. Due to the long-run link between money growth and inflation these deviations are also apparent in filtered measures of money growth. Thus, a central bank that responds to persistent and significant deviations of money growth by adjusting interest rates can effectively offset the policy bias arising from misperceptions about potential output and other unobservables.

Our findings open up several interesting avenues for further research. For example, allowing for unforeseen, permanent shifts in velocity, i.e. shifts in money demand parameters, the information content of long-run money growth would depend on how quickly the central bank learns the new parameter values. Furthermore, we have focused on strict inflation targeting with backward-looking expectations. In this case, cross-checking for persistent shifts is relatively straight-
forward as inflation and adjusted money-growth are expected to be white-noise processes. Extending the analysis to allow for partially forward-looking market expectations would not change this feature of our economy. However, flexible inflation targeting (with the output gap in the central bank’s loss function) would introduce mean reversion in inflation and adjusted money growth dynamics. In this case, a more sophisticated test may be required for cross-checking. Finally, our baseline model may be extended to render filtered money growth a leading indicator of filtered inflation, such that it clearly dominates filtered inflation as the object of cross-checking.

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